SOLUCIONES VVA (Avisar si hay algún error)

1)

$$P(R > 0.2X) = \int_{x=0}^{x=\infty} \int_{r=0.2x}^{r=\infty} f(r)\ f(x)dr\ dx$$

con: $f(r) = f_\gamma(r/1; 0.04) = 0.04e^{-0.04r}$

y $f(x) = f_N(x/100; 30) = \frac{1}{\sqrt{2\pi}30} e^{-\frac{(x-100)^2}{2\ 30^2}}$

2)

(**a**)

$$P(A < 80 \cup B < 80) = 1 - P\,(A \geq 80)\,(B \geq 80) = 1 - \int_{a=80}^{a=\infty} \int_{b=80}^{b=\infty} f(a)f(b)db\ da$$

con: $f(a) = f_N(a/100; 30) = \frac{1}{\sqrt{2\pi}30} e^{-\frac{(a-100)^2}{2\ 30^2}}$

y $f(b) = f_\gamma(b/4; \frac{1}{15}) = \frac{1/15}{3!} \left(\frac{b}{15}\right)^3 e^{-\frac{b}{15}}$

o, también $= 1 - \left[1 - F_z(\frac{80-100}{30})\right] \left[1 - F_\gamma(80/4; \frac{1}{15})\right]$

(**b**)

$$P(A < B) = \int_{a=0}^{a=\infty} \int_{b=a}^{b=\infty} f(a)f(b)db\ da$$

3)

$$P(\frac{C}{10} < T) = \int_{c=0}^{c=\infty} \int_{t=c/10}^{t=\infty} f(c)f(t)dt\ dc$$

con: $f(c) = f_N(c/100; 25) = \frac{1}{\sqrt{2\pi}25} e^{-\frac{(c-100)^2}{2*30^2}}$

y $f(t) = f_\gamma(4; 0.5) = \frac{0.5}{3!}\ (0.5t)^3\ e^{-0.5t}$

4)

$X \sim G(1; 0.05) \quad Y \sim G(1; 0.025)$

$P(Y < X/X + Y < 30) = \frac{P(Y<X)(X+Y<30)}{P(X+Y<30)}$

donde: $P(X + Y < 30) = \int_{y=0}^{y=30} \int_{x=0}^{x=30-y} f(x)f(y)dx\ dy$

$$P(Y < X)(X + Y < 30) = \int_{y=0}^{y=15} \int_{x=y}^{x=30-y} f(x)f(y)dx\ dy$$

con: $f(x) = 0.05e^{-0.05x}$

$f(y) = 0.025e^{-0.025y}$

5)

$X \sim N(110; 30) \quad Y \sim N(90; 25)$

(**a**)

$P(X < Y/X+Y < 230) = \frac{P(X<Y)(X+Y<230)}{P(X+Y<230)}$

donde: $P(X+Y < 230) = \int\limits_{x=0}^{x=230}\int\limits_{y=0}^{y=230-x} f(x)f(y)dy\ dx$

$$P(X < Y)(X+Y < 230) = \int\limits_{x=0}^{x=115}\int\limits_{y=x}^{y=230-x} f(x)f(y)dy\ dx$$

con: $f(x) = \frac{1}{\sqrt{2\pi}30}e^{-\frac{(c-110)^2}{2\ 30^2}}$

$f(y) = \frac{1}{\sqrt{2\pi}25}e^{-\frac{(c-90)^2}{2\ 25^2}}$

(**b**)

$P(\pi) = P(X+Y < 230)$ (anteriormente calculado)

$f^T(x;y) = \frac{f(x)f(y)}{P(\pi)}$

$f^*(y) = \int\limits_{x=0}^{x=230-y} f^T(x;y)dx$ para $0 < y < 230$

6) ---

$P(\pi) = P(B^2+H^2 > 4^2)$

$= 1 - P(B^2+H^2 \leq 4^2) = 1 - \int\limits_{b=0}^{b=4}\int\limits_{h=0}^{h=\sqrt{4^2-b^2}} \frac{2}{100}b\ \frac{1}{5}dh\ db$

$f^T(b;h) = \frac{f(b)f(h)}{P(\pi)} = \frac{2}{100}b\ \frac{1}{5}/P(\pi)$

$$f^*(b) = \begin{cases} \int\limits_{h=\sqrt{4^2-b^2}}^{h=5} f^T(b;h)dh & \text{para } 0 < b < 4 \\ \int\limits_{h=0}^{h=5} f^T(b;h)dh & \text{para } 4 \leq b < 10 \end{cases}$$

7) ---

(**a**) primero: X segundo: Y

$P(X > Y+3) = \int\limits_{x=3}^{x=10}\int\limits_{y=0}^{y=x-3} \frac{2x}{100}\frac{2y}{100}dy\ dx$

(**b**)

$P(\pi) = P(X > Y+3)$

$f^T(x;y) = \frac{2x}{100}\frac{2y}{100}/P(\pi)$

$f^*(y) = \int\limits_{x=y+3}^{x=10} f^T(x;y)dx$ para $0 < y < 7$

8) ---

$$\int_{r=2}^{r=9}\int_{h=1}^{h=10-r} k(r+h)dh\ dr = 1 \quad \text{sale } k.$$

(**a**)

$$\mu_V = E(\pi R^2 H) = \int_{r=2}^{r=9}\int_{h=1}^{h=10-r} \pi r^2 h\ k(r+h)dh\ dr$$

(**b**)

$$P(H<5) = \int_{h=1}^{h=5}\int_{r=2}^{r=10-h} k(r+h)dr\ dh$$

(**c**)

$P(\pi) = P(H<5)$

$f^T(r;h) = \frac{k(r+h)}{P(\pi)}$

$$f^*(r) = \begin{cases} \int_{h=1}^{h=5} f^T(r;h)dh & \text{para } 2<r<5 \\ \int_{h=1}^{h=10-r} f^T(r;h)dh & \text{para } 5\le r<9 \end{cases}$$

9)

(**a**)

$$\int_{r=0}^{r=5}\int_{h=5-r}^{h=10-2r} kh^2(1+r)dh\ dr = 1 \text{...sale } k.$$

(**b**)

$P(\pi) = P(H>2R)$

$$= \int_{h=3.33}^{h=5}\int_{r=5-h}^{r=h/2} kh^2(1+r)dr\ dh + \int_{h=5}^{h=10}\int_{r=0}^{r=(10-h)/2} kh^2(1+r)dr\ dh$$

$f^T(r;h) = \frac{kh^2(1+r)}{P(\pi)}$

$$\mu_V^T = E(\pi R^2 H) = \int_{h=3.33}^{h=5}\int_{r=5-h}^{r=h/2} \pi r^2 h\ f^T(r;h)dr\ dh + \int_{h=5}^{h=10}\int_{r=0}^{r=(10-h)/2} \pi r^2 h\ f^T(r;h)dr\ dh$$

(**c**)

$$f^*(h) = \begin{cases} \int_{r=5-h}^{r=h/2} f^T(r;h)dr & \text{para } 3.33<h<5 \\ \int_{r=0}^{r=(10-h)/2} f^T(r;h)dr & \text{para } 5<h<10 \end{cases}$$

10)

$X \sim N(130;20) \quad Y \sim G(1;0.01)$

$f(x;y) = f(x)f(y) = \frac{1}{\sqrt{2\pi}20} e^{-\frac{(x-130)^2}{2\ 20^2}} 0.01e^{-0.01y}$

(**a**)

$$P(X < Y) = \int_{x=0}^{x=\infty} \int_{y=x}^{y=\infty} f(x;y)dy\ dx$$

(**b**)

$P(\pi) = P(X < Y)$

$f^T(x;y) = \frac{f(x;y)}{P(\pi)}$

$$f^*(x) = \int_{y=x}^{y=\infty} f^T(x;y)dy \qquad \text{para } x > 0.$$

11)

$$\int_{r=0}^{r=10} \int_{h=0}^{h=10-r} cte\ r(r^2+h^2)dh\ dr = 1 \text{....sale } cte.$$

(**a**)

$$P(\pi) = P(\pi R^2 H > 150) = \int_{r=2.528}^{r=9.467} \int_{h=150/\pi r^2}^{h=10-r} cte\ r(r^2+h^2)dh\ dr$$

$f^T(r;h) = \frac{cte\ r(r^2+h^2)}{P(\pi)}$

$$\mu_V = E(\pi R^2 H) = \int_{r=2.528}^{r=9.467} \int_{h=150/\pi r^2}^{h=10-r} \pi R^2 H\ f^T(r;h)dh\ dr$$

(**b**)

$$f^*(r) = \int_{h=150/\pi r^2}^{h=10-r} f^T(r;h)dh \qquad \text{para } 2.528 < r < 9.467$$

12)

$f(r) = \frac{1}{2}$ para $5 < r < 7$ $\qquad f(y) = \frac{1}{\sqrt{2\pi}300} e^{-\frac{(y-1100)^2}{2\ 300^2}}$

(**a**)

$$P(Y > \pi R^2 15) = \int_{r=5}^{r=7} \int_{y=\pi r^2 15}^{y=\infty} f(r)f(y)dy\ dr$$

(**b**)

$$R = \begin{cases} 0 & para\ \ Y \le \pi R^2 15 \\ Y - \pi R^2 15 & para\ \ Y > \pi R^2 15 \end{cases}$$

$$\mu_R = E(R) = \int_{r=5}^{r=7} \int_{y=0}^{y=\pi r^2 15} 0\ f(r)f(y)dy\ dr + \int_{r=5}^{r=7} \int_{y=\pi r^2 15}^{y=\infty} (y - \pi r^2 15) f(r)f(y)dy\ dr$$

(**c**)

$$H = \begin{cases} Y/\pi R^2 & para\ \ Y \le \pi R^2 15 \\ 15 & para\ \ Y > \pi R^2 15 \end{cases}$$

$$\mu_H = E(H) = \int_{r=5}^{r=7}\int_{y=0}^{y=\pi r^2 15} \tfrac{y}{\pi r^2}\ f(r)f(y)dy\ dr + \int_{r=5}^{r=7}\int_{y=\pi r^2 15}^{y=\infty} 15\ f(r)f(y)dy\ dr$$

13)

$f(a) = \frac{1}{\sqrt{2\pi}30}e^{-\frac{(a-100)^2}{2\ 30^2}}$ $\qquad f(b) = \frac{1}{\sqrt{2\pi}20}e^{-\frac{(b-80)^2}{2\ 20^2}}$

(**a**)

$$T = \begin{cases} A & para\ \ A < B \\ B & para\ \ A \geq B \end{cases}$$

$$\mu_T = E(T) = \int_{a=0}^{a=\infty}\int_{b=0}^{b=a} bf(a)f(b)db\ da + \int_{a=0}^{a=\infty}\int_{b=a}^{b=\infty} af(a)f(b)db\ da$$

$$\sigma_T^2 = E(T^2) - \mu_T^2$$
$$= \int_{a=0}^{a=\infty}\int_{b=0}^{b=a} b^2 f(a)f(b)db\ da + \int_{a=0}^{a=\infty}\int_{b=a}^{b=\infty} a^2 f(a)f(b)db\ da - \mu_T^2$$

(**b**)

$P(B < A/A + B > 200) = \frac{P(B<A)(A+B>200)}{P(A+B>200)}$ donde:

$$P(A + B > 200) = 1 - P(A + B \leq 200) = 1 - \int_{a=0}^{a=200}\int_{b=0}^{b=200-a} f(a)f(b)db\ da$$

$$P(B < A)(A+B > 200) = \int_{a=100}^{a=200}\int_{b=200-a}^{b=a} f(a)f(b)db\ da + \int_{a=200}^{a=\infty}\int_{b=0}^{b=a} f(a)f(b)db\ da$$

14)

$f(x; a) = f(x)f(a) = \frac{1}{\sqrt{2\pi}30}e^{-\frac{(x-100)^2}{2\ 30^2}}0.02e^{-0.02a}$

$$T = \begin{cases} X & para\ \ X < A \\ A + \frac{X-A}{2} & para\ \ X \geq A \end{cases} \quad \text{(notar que: } A + \tfrac{X-A}{2} = \tfrac{A+X}{2}\text{)}$$

$$\mu_T = E(T) = \int_{x=0}^{x=\infty}\int_{a=0}^{a=x} \tfrac{a+x}{2} f(x;a)da\ dx + \int_{x=0}^{x=\infty}\int_{a=x}^{a=\infty} xf(x;a)da\ dx$$

$$\sigma_T^2 = E(T^2) - \mu_T^2$$
$$= \int_{x=0}^{x=\infty}\int_{a=0}^{a=x} \left(\tfrac{a+x}{2}\right)^2 f(x;a)da\ dx + \int_{x=0}^{x=\infty}\int_{a=x}^{a=\infty} x^2 f(x;a)da\ dx - \mu_T^2$$

15)

$X \sim G(1; 0.05);\ R \sim G(1; 0.2) \quad f(x; r) = 0.05e^{-0.05x}0.2e^{-0.2r}$

(**a**)

$$E(\boldsymbol{\rho}) = E(\tfrac{X}{X+R}) = \int_{r=0}^{r=\infty}\int_{x=0}^{x=\infty} \tfrac{x}{x+r}\ f(x;r)dx\ dr$$

(**b**)

$P(\boldsymbol{\rho} > 0.9) = P(\frac{X}{X+R} > 0.9) = P(X > 0.9X + 0.9R) = P(0.1X > 0.9R)$

$$= P(X > 9R) = \int\limits_{r=0}^{r=\infty}\int\limits_{x=9r}^{x=\infty} f(x;r)dx\ dr$$

16)

$X \sim N(10;2)$; $T \sim G(3;0.3)$ con: $f(x;t) = \frac{1}{\sqrt{2\pi}2} e^{-\frac{(x-10)^2}{2\ 2^2}} \frac{0.3}{2!}(0.3t)^2 e^{-0.3t}$

(**a**)

$$P(T < X) = \int\limits_{x=0}^{x=\infty}\int\limits_{t=0}^{t=x} f(x;t)dt\ dx$$

(**b**)

$$P(X > 9)(T > 9) = \int\limits_{x=9}^{x=\infty}\int\limits_{t=9}^{t=\infty} f(x;t)dt\ dx$$

(**c**)

$P(\pi) = P(X > 9)(T > 9)$

$f^T(x;t) = \frac{f(x;t)}{P(\pi)}$

(en zona de paso, $L = \begin{cases} T & \text{si } T < X \\ X & \text{si } T > X \end{cases}$)

$$\mu_L = E(L) = \int\limits_{x=9}^{x=\infty}\int\limits_{t=9}^{t=x} t f^T(x;t)dt\ dx + \int\limits_{x=9}^{x=\infty}\int\limits_{t=x}^{t=\infty} x f^T(x;t)dt\ dx$$
